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Phenomenology of Randall–Sundrum Black Holes

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Abstract

We explore the phenomenology of microscopic black holes in the S^1/Z_2 Randall-Sundrum (RS) model. We consider the canonical framework in which both gauge and matter fields are confined to the brane and only gravity spills into the extra dimension. The model is characterized by two parameters, the mass of the first massive graviton (m_1), and the curvature $1/\ell$ of the RS anti-de Sitter space. We compute the sensitivities of present and future cosmic ray experiments to black hole mediated events, for a wide range of ℓ and m_1, and compare them with the sensitivities of Tevatron Runs I and II to higher-dimensional physics. As part of our phenomenological analysis, we examine constraints placed on ℓ by AdS/CFT considerations.

One of the most exciting predictions of sub-millimeter extra dimensions [1, 2] is the production of black holes (BHs) in particle collisions with center-of-mass energy larger than a TeV and sufficiently small impact parameter [3, 4, 5, 6, 7, 8]. Although colliders have not yet attained the energies required to probe this new strong quantum gravitational effect, the extraordinarily high center-of-mass energies achieved at the top of the atmosphere in ultra-high energy cosmic ray collisions are high enough to render any change in spacetime dimensionality detectable [9, 10, 11, 12, 13]. Among these cosmic rays, a nearly guaranteed flux of neutrinos (produced through interactions of extremely high energy protons with the cosmic microwave background) could produce BHs which decay promptly initiating deeply developing air showers far above the Standard Model (SM) rate [9], and with very distinctive characteristics [10]. In addition, neutrinos that traverse the atmosphere may produce BHs through interactions in the ice or water and be detected by neutrino telescopes [13]. Moreover, in scenarios with asymmetric compactifications the production of brane configurations wrapped around small extra dimensions may be competitive with BH production [14]. Very recently, based on the absence of a significant signal of deeply developing showers reported by the AGASA Collaboration we derived new limits on the fundamental Planck scale [12]. In this paper we expand upon this study and examine in more detail the phenomenological implications of BH production in the Randall–Sundrum (RS) scenario [2].

The RS model consists of two 3-branes (with equal and opposite tensions $\sigma_{\rm Planck} = -\sigma_{\rm SM} = 12\, M^3/\ell$) which rigidly reside at S^1/Z_2 orbifold fixed points at the boundaries ($y = 0$ and $y = \pi r_c$) of a slab of anti-de Sitter (AdS) space of radius ℓ. The classical action describing the system is given by [2]

$$S = M^3 \int d^4x \int_0^{\pi r_c} dy \sqrt{-g}\, \left(\frac{12}{\ell^2} + R\right) + \int d^4x \sqrt{-\eta}\; \sigma_{\rm SM} + \int d^4x \sqrt{-\eta}\; \sigma_{\rm Planck}\,, \quad (1)$$

where R is the 5-dimensional Ricci scalar in terms of the metric $g_{\mu\nu}$, M is the fundamental scale of gravity, and η_{ij} is the flat Minkowskian metric. In what follows, the Latin subscripts extend over ordinary 4-dimensional spacetime, whereas Greek subscripts represent all 5 dimensions. The metric satisfying this Ansatz (in horospherical coordinates) reads

$$ds^2 = e^{-2|y|/\ell}\; \eta_{ij}\, dx^i dx^j + dy^2\,. \quad (2)$$

Examination of the action in the 4-dimensional effective theory leads to [2]

$$\overline{M}_{\rm Pl}^2 = M^3\, \ell\, \left(1 - e^{-2\,\pi\, r_c/\ell}\right)\,, \quad (3)$$

where $\overline{M}_{\rm Pl}$ is the reduced effective 4-dimensional Planck scale. Now, assuming that SM fields are localized on the 3-brane at $y = \pi\, r_c$, one finds that a field with the fundamental mass parameter m_0 will appear to have the physical mass $m = e^{-\pi\, r_c/\ell}\, m_0$. Hence, TeV scales can be generated from fundamental scales of order $M_{\rm Pl}$ through the exponential warping factor. Specifically, the observed hierarchy between the gravitational and electroweak mass scales is reproduced provided $r_c/\ell \approx 12$. The 4-dimensional phenomenology of this model (only gravity propagates into the bulk) is governed by two parameters: $c = (\ell \overline{M}_{\rm Pl})^{-1}$, which is expected to be near though somewhat less than unity, and m_1 which is the mass of the first Kaluza–Klein graviton excitation [15].

Two different types of BHs can be produced in trans-Planckian particle collisions within this set up: (i) AdS/Schwarzschild BHs that propagate freely into the bulk (generally falling towards the AdS horizon once produced) and (ii) tubular pancake shape BHs that are bound to the brane [5, 16]. To study the phenomenology of the latter, it is convenient to define a new variable $z = \ell\, e^{y/\ell}$. In such a coordinate system the metric

$$ds^2 = \frac{\ell^2}{z^2}\ \left(dz^2 + \eta_{ij}\, dx^i\, dx^j\right) \tag{4}$$

is conformal to a 5-dimensional flat metric. To describe the SM brane we introduce the coordinate $w = z - z_c$, where $|w| \in (0, w_c)$, $w_c = \ell\,(e^{\pi r_c/\ell} - 1)$, and the TeV brane is located at $w = 0$. After a conformal redefinition

$$g_{\mu\nu} \equiv \left(\frac{\ell}{z_c + w}\right)^2 \tilde{g}_{\mu\nu}\,, \tag{5}$$

one obtains [17]

$$R = \left(\frac{z_c + w}{\ell}\right)^2\ \tilde{R} - \frac{20}{\ell^2}\,, \tag{6}$$

where $\tilde{R}$ is the Ricci scalar calculated with the metric $\tilde{g}_{\mu\nu}$. Initially Minkowskian, the metric $\tilde{g}$ will be modified, under conditions to be delineated, to include as a patch the 5-dimensional Schwarzschild solution. The gravity sector can now be rewritten as

$$\begin{aligned} S_{\rm gravity} &= M^3 \int d^4x \int_0^{w_c} dw\,\sqrt{-\tilde{g}}\ \left(\frac{\ell}{z_c + w}\right)^3 \left(\tilde{R} - \frac{8}{(z_c + w)^2}\right) \\ &\simeq \widetilde{M}^3 \int d^4x \int_0^{w_{max} \ll z_c} dw\,\sqrt{-\tilde{g}}\ \left(\tilde{R} - \frac{8}{z_c^2}\right)\,, \end{aligned} \tag{7}$$

where

$$M \equiv \widetilde{M} z_c/\ell\ \ . \tag{8}$$

It is worth emphasizing that $\widetilde{M}$ refers to the canonical frame, and is expected to be of order 1 TeV.

We will be interested in the domain of parameter space for which a high energy collision as viewed from the SM brane can result in the formation of a 5-dimensional spherical flat space black hole. A calculation of metric perturbations due to a source on the $w = 0$ brane can be made in the flat 5-dimensional space-time approximation, *i.e.,* ignoring the effects of the AdS term in Eq. (7), if $w \ll z_c$ [18], where z_c is the AdS curvature as viewed on the SM brane, with a gravitational constant $\widetilde{M}$. This implies that to use flat space BH formulae we must require $\tilde{r}_s \ll z_c$, where $\tilde{r}_s$ denotes the size of 5-dimensional Schwarzschild radius in the canonical frame [19]

$$\tilde{r}_s(\widetilde{M}_{\rm BH}) = \sqrt{\frac{2\,\widetilde{M}_{\rm BH}}{3\,\pi\,\widetilde{M}_D^3}}\,, \tag{9}$$

where $\widetilde{M}_{\rm BH} = \sqrt{\tilde{s}}$ is the BH mass, s is the center-of-mass energy in terms of $\tilde{g}_{\mu\nu}$, and $\widetilde{M}_D = (4\pi)^{1/3}\,\widetilde{M}$.[1] From Eqs. (3), (8), and the forgoing relation between $\widetilde{M}_D$ and $\widetilde{M}$ we

[1] We set $M_D^3 = (4\,G_5)^{-1}$, where G_5 is the 5-dimensional Newton constant.

find that the condition $\tilde{r}_s < z_c$ leads to an *upper* bound on the mass of the black hole for which this picture is valid [20]:

$$\widetilde{M}_{\rm BH}/\widetilde{M}_D < 24\ c^{-4/3} \quad , \tag{10}$$

where, once more, $c \equiv \left(\ell \overline{M}_{\rm Pl}\right)^{-1}$. When the energy exceeds this bound, the behavior of the cross section may be analyzed within the AdS/CFT dual picture [21], and may assume the $\ln^2 E$ behavior conforming to the Froissart bound [22].

As mentioned earlier, the parameter c is expected to be small. Comparison of the RS brane tension and the D-brane tension in perturbative heterotic string theory suggests that $c \lesssim 0.1$ [15]. Work in recent years linking the Randall-Sundrum brane-world mechanism to the conjectured AdS/CFT correspondence [21] allows an independent and similar bound on c. The one-sided brane world relation, Eq.(3) with $r_c \to \infty$, is equivalent to

$$c = \left(\ell \overline{M}_{\rm Pl}\right)^{-1} = (M\ell)^{-3/2} \quad . \tag{11}$$

In this "single brane" limit [23], Duff and Liu [24] pointed out a complementarity between AdS_5 and an $\mathcal{N} = 4$ superconformal $\mathrm{U}(N)$ Yang-Mills theory living on the four-dimensional brane (boundary): corrections to Newton's law from both sectors are equal when the AdS/CFT Weyl anomaly relation [25],

$$2G_5 N^2 = \pi \ell^3 \,, \tag{12}$$

or

$$(M\ell)^3 = N^2/(8\pi^2) \tag{13}$$

holds. For finite but large r_c, deviations from conformality are exponentially damped in the infrared [26]. BH creation in SM brane collisions becomes significant if a weak-gravity 5-dimensional description is valid [27]. This occurs for $M\ell \gg 1$, and hence $c \ll 1$. A more quantitative estimate emerges by first combining Eqs. (11) and (13) to obtain

$$c = 2\sqrt{2}\pi/N \quad . \tag{14}$$

An approximate lower bound on N may be surmised by noting that the KK gravitons find a dual description as glueballs resulting from strong coupling in the (approximate) CFT sector [27]. In the large-N analysis of 't Hooft [28], this requires that the planar loop expansion parameter $g^2_{\rm YM} N/8\pi^2 > 1$. For $g^2_{\rm YM}/4\pi \simeq 0.1$, we obtain from (14)

$$c \lesssim 0.1 \tag{15}$$

The key question now is at what mass ratio $\left(\widetilde{M}_{\rm BH}/\widetilde{M}_D\right)$ is the BH description valid. According to the semiclassical prescription, the BH evaporation is governed by its Hawking temperature

$$\tilde{T}_H = \frac{1}{2\,\pi\,\tilde{r}_s} \,. \tag{16}$$

Since the wavelength $\tilde{\lambda} = 2\pi/\tilde{T}_H$ corresponding to this temperature is larger than the BH size, to a very good approximation the BH behaves like a point-radiator with entropy

$$\tilde{S} = \frac{4}{3}\,\pi\,\widetilde{M}_{\rm BH}\,\tilde{r}_s = \sqrt{\frac{32\pi}{27}}\,\left(\frac{\widetilde{M}_{\rm BH}}{\widetilde{M}_D}\right)^{3/2} \quad . \tag{17}$$

The magnitude of the entropy indicates the validity of this picture. Thermal fluctuations due to particle emission are small when $\tilde{S} \gg 1$ [29], and statistical fluctuations in the microcanonical ensemble are small for $\sqrt{\tilde{S}} \gg 1$ [6]. In searches for BH mediated events at colliders, it is essential to set $x_{\rm min} \equiv \widetilde{M}_{\rm BH}^{\rm min}/\widetilde{M}_D$ high enough that the decay branching ratios predicted by the semiclassical picture of BH evaporation are reliable. The QCD background is large, and therefore the extraction of signal from background at hadron colliders depends on knowing the BH decay branching ratios reliably. This is especially true if one is attempting to determine discovery limits, where the overall rates for BH production are not necessarily large. Thus, in collider searches, a cutoff of $x_{\rm min} = 5.5$ (i.e., $\tilde{S} > 25$) or more may be appropriate. By contrast, the search for deeply penetrating quasi-horizontal showers initiated by BH decays can afford to be much less concerned with the details of the final state, since the background is almost nonexistent. As a result, the signal relies only on the existence of visible decay products, which, in this context, includes all particles other than neutrinos, muons, and gravitons. Indeed, there is very little about the final state, other than its total energy and to some degree its multiplicity and electromagnetic component [10], that we can reasonably expect to observe, since detailed reconstruction of the primary BH decay process is not possible at cosmic ray detectors. It seems reasonable to choose a significantly lower value of $\widetilde{M}_{\rm BH}^{\rm min}$ than is needed for collider searches; in our estimates of rates for cosmic ray facilities we will take $x_{\rm min}$ as low as 1, or $\tilde{S}$ as low as 2. While BHs of mass around $\widetilde{M}_D$ will be outside the semiclassical regime, it seems quite reasonable to expect that they will nevertheless decay visibly, whatever 5-dimensional quantum gravitational description applies. Finally, as a consequence of Eq.(10) there are upper bounds on the entropy. Combining the latter with Eq. (17) we obtain

$$\tilde{S} < 250 \; c^{-2} \quad . \tag{18}$$

To model the collision between two partons i and j, with center-of-mass energy $\sqrt{\hat{s}}$ and impact parameter $\tilde{b} \lesssim \tilde{r}_s$, we consider a scattering amplitude of an absorptive black disk with area $\pi\, \tilde{r}_s^2$ [3]. Criticisms [30] of the assumptions leading to these cross sections have been addressed [31]. Additionally, when considering non-zero impact parameters in 5 dimensions, one must be sure that the production of BHs is favored over that of black rings [32] for the relevant values of the angular momentum J. For a 5-dimensional Kerr solution [19] with a single angular momentum J (taken in the direction of the brane [6]), the horizon size r_k and J for a mass $M_{\rm BH}$ are related:

$$\frac{4}{9}\, r_k^2 + \frac{J^2}{M_{\rm BH}^2} = \frac{32 G_5 M_{\rm BH}}{27\pi} \quad . \tag{19}$$

A geometric cross section requires absorption in angular momenta up to $J \simeq M_{\rm BH} r_k/2$. On incorporating this condition in (19) one finds that the maximum value of J required for the geometric cross section is $0.83\, M_{\rm BH}\, \sqrt{32\, G_5\, M_{\rm BH}/27\pi}$. This is below the *lower* limit on the spin of a black ring, $0.92\, M_{\rm BH}\, \sqrt{32\, G_5\, M_{\rm BH}/27\pi}$ [32]. Therefore, it is consistent to consider only the production of 5-dimensional BHs with geometric cross section. For the same reasons as discussed in [12], we will calculate our cross sections with the Schwarzschild radius.

In our investigation of BH production by cosmic rays, we will be most interested in collisions of neutrinos with atmospheric nucleons. In order to obtain the νN cross section we take the geometric cross section $\hat{\sigma}$, fold in the appropriate parton densities and integrate

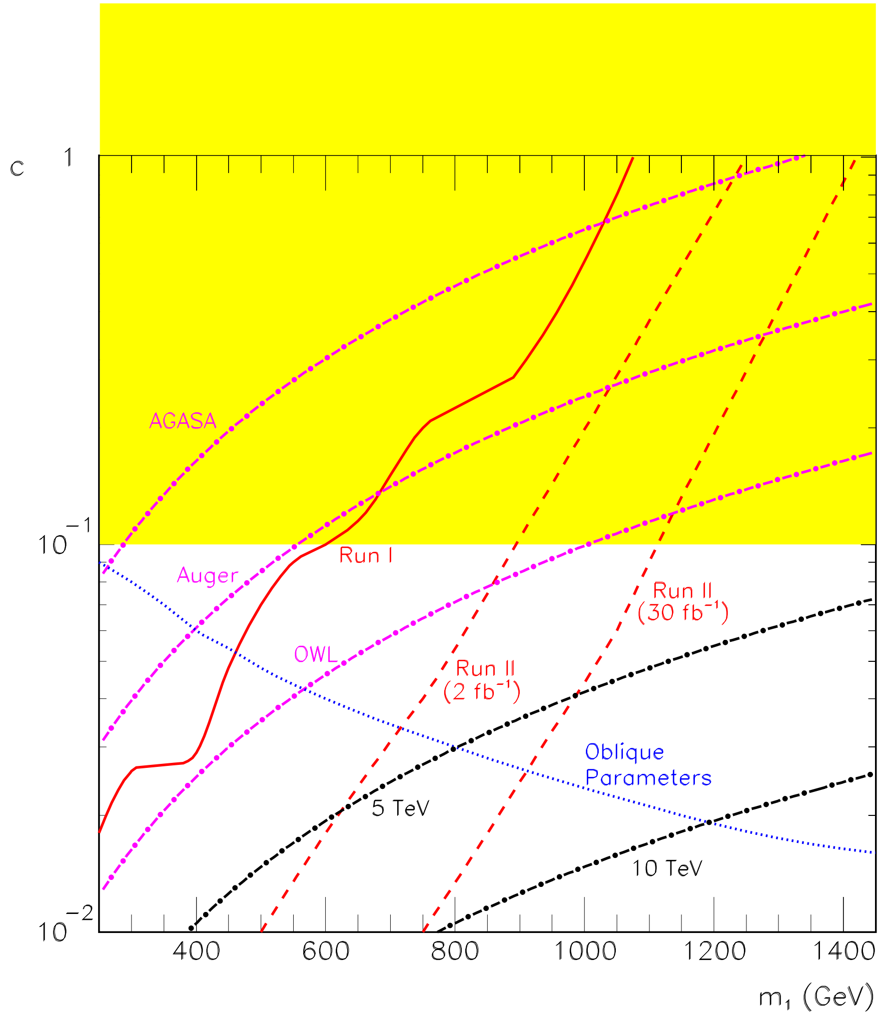

FIG. 1: Allowed region of the RS parameter space. Resonant production of the first KK graviton excitation in the Drell-Yan and dijet channels at the Tevatron [43] has excluded the region to the left of the solid bumpy curve [15], whereas an analysis of the oblique parameters excludes the region below the smoothly falling curve (dotted) [44]. The diagonal dashed lines correspond to future 95% CL parameter exclusion region at Run II at the Tevatron under the assumption that no signal is found [15]. The smoothly (dashed-dotted) rising curves indicate contours of constant $\widetilde{M}_D$ in the $c - m_1$ plane. The region above AGASA's curve is excluded at 95% CL. For an observation time of 5 (3) years, Auger (OWL) will probe the region above its curve. The unshaded area indicates the region delineated by Eq.(15).

over the parton energies [9]

$$\sigma(\nu N \to \text{BH}) = \sum_i \int_{(\widetilde{M}_{\rm BH}^{\rm min})^2/s}^1 dx\, \hat{\sigma}_i(\sqrt{xs})\, f_i(x,Q) \;, \tag{20}$$

where $s = 2m_N E_\nu$, the sum is over all partons in the nucleon, and the f_i are parton distribution functions. We set the momentum transfer $Q = \min\{\widetilde{M}_{\rm BH}, 10 \text{ TeV}\}$, where the upper limit is from the CTEQ5M1 distribution functions [33]. Also, as a consequence of Eq. (10), there will be an upper bound on the parton subenergies. However, the effect is negligible over virtually the entire region of c.

With this in mind, for an incoming neutrino flux $d\Phi/dE_\nu$, the event rate for deep showers is

$$\mathcal{N} = \int dE_\nu \, N_A \, \frac{d\Phi}{dE_\nu} \, \sigma(\nu N \to \mathrm{BH}) \, A(E_\nu) \, T \ , \tag{21}$$

where $N_A = 6.022 \times 10^{23}$ is Avogadro's number, $A(E_\nu)$ is the acceptance for quasi-horizontal showers in cm^3 water-equivalent steradians, and T is the experiment running time. In what follows we adopt the cosmogenic neutrino flux estimates of Protheroe and Johnson [34], assuming an energy cutoff in the injection spectrum at $E_{\rm cutoff} = 3 \times 10^{21}$ eV. This is in very good agreement with the most recent evaluation of the cosmogenic neutrino flux assuming a strong cosmological evolution on the cosmic ray sources, which scales like $\propto (1+z)^4$ for redshift $z < 1.9$ and becomes flat at higher redshifts [35]. To explore possible additional contributions from semi-local nucleon sources, we also consider below the cosmogenic neutrino flux estimates of Hill and Schramm [36]. Bounds on $\widetilde{M}_D$ emerge as a result of adopting the AGASA limit of 1 possible event observed, with a background of 1.72 events [37]. This places a limit of ≤ 3.5 events at 95% CL [38]. Inserting our cross section into Eq. (21) and requiring $\mathcal{N} \leq 3.5$ leads to exclusion of the region $\widetilde{M}_D \lesssim 0.70 \pm 0.05$ TeV at 95% CL [12]. The variation in this bound reflects uncertainty in the neutrino flux. The forthcoming facilities of the Auger Observatory [39] will probe fundamental Planck scales, $\widetilde{M}_D \lesssim 1.55 \pm 0.15$ TeV [12], assuming 5 years of data and no excess above SM neutrino background. The variation in this case includes uncertainties in the hadronic background and the experimental aperture.

Beyond Auger, NASA has authorized studies for the satellite cosmic ray detection facility known as the Orbiting Wide-angle Light-collectors (OWL) [40], projected for 2007. In a comprehensive recent study, BH event rates at OWL for $n \geq 2$ large extra dimensions were reported [41]. The greatly increased aperture of this space-based facility will allow the detection ~ 10 BH/yr for $x_{\rm min}$ as large as 5. This is well into the region of large entropy, permitting comparison with characteristic shower profiles. In the case of present interest ($n = 1$), the "eyes of the OWL" will substantially extend the region of the $c - m_1$ plane probed in cosmic ray observations. Under the assumption that the cosmic ray sources have a strong cosmological evolution, the expected background from SM neutrino interactions (all flavors) is 3 events/year [41]. On the basis of zero hadronic background, three years of observation implies a deviation from the SM at a 95% CL for 7.77 events observed above background [38]. If these are BHs produced with $x_{\rm min} = 1$, this represents a sensitivity to $\widetilde{M}_D \lesssim 2.8$ TeV at the 95% CL [42].

Summary. The Randall-Sundrum model predicts a series of TeV-scale graviton resonances with weak scale couplings to SM fields: this makes the Tevatron an outstanding probe of RS physics. Analysis [15] of Tevatron data [43] for anomalous Drell-Yan and dijet production as well as the calculation of indirect contributions to electroweak observables [44] already place significant constraints on the $c - m_1$ plane. In this work we have developed the formalism for calculating RS black hole production through cosmic ray neutrino collisions in the atmosphere, and have delineated additional constraints placed on the RS model by existing and future cosmic ray data. The complete allowed region is summarized in Fig. 1. The lines of constant $\widetilde{M}_D$ reflect the relation $m_1 = 3.83\ c^{2/3}\ (4\pi)^{-1/3} \widetilde{M}_D$. AGASA is able only marginally to probe the parameter space allowed by the Tevatron in the limited region $m_1 \gtrsim 1$ TeV and $c > 0.6 - 0.7$. Five years of running at Auger can reveal events in the region $m_1 > 1$ TeV and $c > 0.5$ which will not be tested by the Tevatron Run II A. A year of observation by OWL will probe a substantially larger part of the parameter space for small c (see Fig. 1). We have also delineated in our figure the range of the parameter c (related to the AdS radius) favored through AdS/CFT considerations.

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